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Is protected volume of ESE lightning rods really as large as it is declared by French standard NF C 17-102?

The main aim of this article is to show that French standard NF C 17-102 and other national standards based on the French standard don't use rolling sphere method (RSM) properly in case of determining the protected volume of ESE lightning rod. The article doesn't doubt early streamer emission efficiency Δt of ESE lightning rod, but it is trying to prove that calculation of the ESE lightning rod protection radius in any height R_P, according to standards mentioned above, is incorrect.

Key words: standard NF C 17-102, ESE lightning rod, conventional lightning rod, rolling sphere method, protected volume, protection radius

1 INTRODUCTION

Rolling sphere method (also known as electro-geometrical method) is the most universal method for estimating the location of lightning conductors and for determining their protected volume.

Critical distance between downward stepped leader tip and grounded structure at the moment of upward connecting discharge initiation is called striking distance. The distance depends on charge on the leader channel which is related to the peak current of the prospective return stroke. Once the charge on the leader channel increases, the striking distance grows too due to increase of return stroke peak current. If the striking distance with starting point in the downward leader tip is assumed to be constant, all attractive striking points can be determined. The striking points can be formed to the spherical region in the three-dimensional space. The spherical region represents rolling sphere with center on the leader tip and with radius calculated as follows [3][5]:

$$r = 10 \cdot I^{0,65} \tag{1}$$

where:

r is radius of rolling sphere [m]

I is prospective return stroke peak current [kA]

The part of the structure that intersects the spherical region of downward leader is the part which will be struck by lightning. This assumption is used when we are trying to find the suitable location for lightning conductors. During that process the downward leader tip is moving in all directions (over coordinates X, Y, Z in three-dimensional space) in a way that spherical region with known radius is touching the protected structure. Movement of the leader tip represents rolling of the sphere over the structure. All prospective strike points of the structure can be determined this way. Once the strike points are determined, lightning conductors can be placed into those points, so that the rolling sphere is touching only lightning conductors, not the structure. The lightning conductors create protected volume which depends on the rolling sphere radius r and on the type of lightning rod (convectional lightning rod or ESE lightning rod).

The protected volume of lightning conductors should be determined for the worst case – the smallest protected volume. This case can be achieved by consideration of the lowest return stroke peak current in equation (1) (Tab. 1):

Tab. 1 Rolling sphere radius depending on lightning protection level LPL [3][4]

LPL	Minimum return stroke peak current I [kA]	Rolling sphere radius r [m]
I	3	20
II	5	30
III	10	45
IV	16	60

2 MATHEMATICAL DERIVATION OF PROTECTION RADIUS OF CONVENTIONAL LIGHTNING ROD

Conventional lightning rod is a simple metal rod without any electronics located under rod tip. Upward leader can be initiated from the rod tip according to pressure, moisture, temperature and electric field intensity. Once the protected volume of the rod is estimated according to [4], no early streamer emission efficiency is taken into consideration.

In this article conventional lightning rod is represented by simple metal rod. The protected volume of single rod determined by rolling sphere method consists only of one three-dimensional volume which is defined by (Fig. 1) :

1. points of touch of rolling sphere with ground (point A_1, A_2) and with rod tip (point B_1),
2. radius of rolling sphere r (Tab. 1).

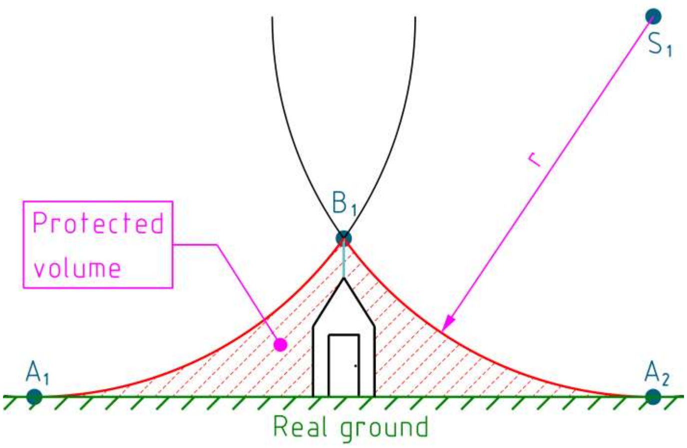

Fig. 1 Cross-section (plane X-Z) of protected volume of single conventional lightning rod determined by rolling sphere method

For better illustration it is suitable to replace protected structure and conventional rod with single rod of height equal to the sum of protected structure height and height of conventional rod (Fig. 2).

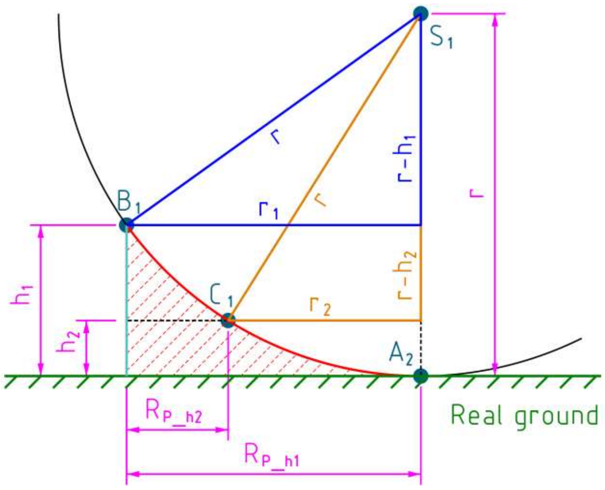

Fig. 2 Mathematical derivation of protection radius of conventional rod R_P

The protected volume of single conventional rod can be replaced by circles in X-Y plane (three-dimensional space) with radius R_P which depends on height h (Z coordinate) above the real ground. This radius can be derived according to Fig. 2 as follows:

1. real ground level (height h_1) - R_{P_h1}:

$$r = \sqrt{r_1^2 + (r - h_1)^2} \tag{2a}$$

$$r_1 = \sqrt{r^2 - (r - h_1)^2} = \sqrt{r^2 - (r^2 - 2 \cdot r \cdot h_1 + h_1^2)} = \sqrt{h_1 \cdot (2 \cdot r - h_1)} \tag{2b}$$

$$R_{P\_h1} = r_1 = \sqrt{h_1 \cdot (2 \cdot r - h_1)} \tag{2c}$$

where:

R_{P_h1} is protection radius in real ground level [m]

r_1 is horizontal distance between center of rolling sphere S1 and rod tip B1 in height h_1 [m]

h_1 is total height of rod tip above the ground [m]

2. any height above real ground level (for instance height h_2) - R_{P_h2}:

$$r = \sqrt{r_2^2 + (r - h_2)^2} \tag{3a}$$

$$r_2 = \sqrt{r^2 - (r - h_2)^2} = \sqrt{r^2 - (r^2 - 2 \cdot r \cdot h_2 + h_2^2)} = \sqrt{h_2 \cdot (2 \cdot r - h_2)} \tag{3b}$$

$$R_{P\_h2} = R_{P\_h1} - r_2 = R_{P\_h1} - \sqrt{h_2 \cdot (2 \cdot r - h_2)} \tag{3c}$$

where:

R_{P_h2} is protection radius in height h_2 [m]

r_2 is horizontal distance between center of rolling sphere S1 and point C1 in height h_2 [m]

h_2 is any height above the ground [m]

The equations (2c), (3c) can only be used under the conditions shown below:

- flat terrain,
- maximal height of rod tip above the ground can't be bigger than radius of rolling sphere considered according to Tab. 1 ($h_{1\_max} \leq r$).

3 MATHEMATICAL DERIVATION OF PROTECTION RADIUS OF ESE LIGHTNING ROD

The ESE lightning rod is conventional rod with electronics added under the rod tip. The main aim of the electronics is to make an earlier initiation of upward leader from the rod tip which makes the leader longer. Length increase of the leader can be calculated using this formula [1]:

$$\Delta L = \Delta T \cdot v \tag{4}$$

where:

ΔL is length increase of upward leader [m]

ΔT is early streamer emission efficiency of ESE rod [µs]

v is speed of upward leader [m/µs]

Length increase of upward leader is assumed to be constant according to [1]. This is because constant speed of upward leader ($v = 1\text{m}/\mu\text{s}$) and constant early streamer emission efficiency of ESE rod given by manufacturer is taken into consideration.

All points in three-dimensional space located in constant length ΔL from the rod tip create additional protection sphere around the tip. This is why total protected volume of ESE rod is larger than the volume of conventional rod. Based on that, the total protected volume of ESE rod determined by rolling sphere method should consist of two parts (Fig. 3):

1. protected spherical volume defined by ([1] doesn't take it into consideration):
   - center in the rod tip of ESE rod (point S0),
   - radius which equals ΔL,
2. protected volume defined by:
   - points of touch of rolling sphere with ground (point A1, A2) and with protected spherical volume (point B1, B2),
   - radius of rolling sphere r (Tab. 1).

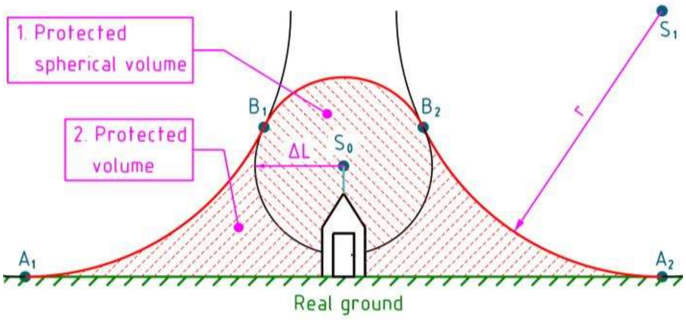

Fig. 3 Cross-section (plane X-Z) of expected protected volume of single ESE lightning rod determined by rolling sphere method

If zero early streamer emission efficiency of ESE rod is taken into consideration ($\Delta L = 0\text{m}$), protected spherical volume should disappear and total protected volume of ESE rod should be equal to the volume of protection of conventional rod according to Fig. 1 (points B_1, B_2 will move to point S_0 and points A_1, A_2 will move towards the protected structure – Fig. 3). The standard [1] doesn't respect that fact and the result is different volume of protection when the zero early streamer emission efficiency of the ESE lightning rod is taken into account.

3.1 DERIVATION OF PROTECTION RADIUS OF ESE LIGHTNING ROD ACCORDING TO NF C 17-102

The ESE lightning rod and protected structure will be again replaced by single rod of height equal to the sum of protected structure height and height of ESE rod (Fig. 3, Fig. 4).

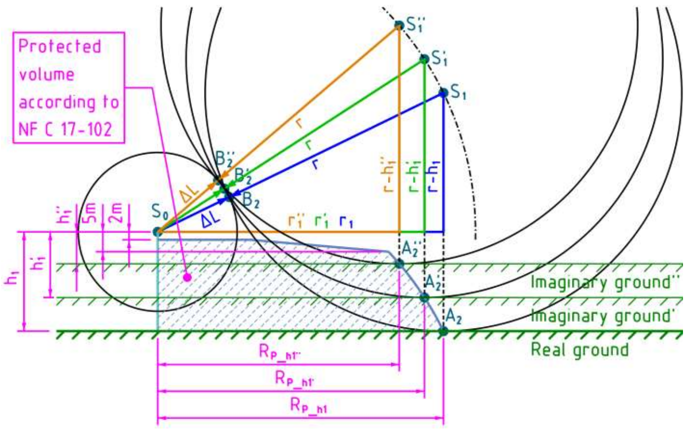

Fig. 4 Cross-section (plane X-Z) of protected volume of single ESE lightning rod determined by rolling sphere method according to [1]

Equation of protection radius shown in [1] is determined as detailed below:

$$r + \Delta L = \sqrt{r_1^2 + (r - h_1)^2} \tag{5a}$$

$$r_1 = \sqrt{(r + \Delta L)^2 - (r - h_1)^2} = \sqrt{(r^2 + 2 \cdot r \cdot \Delta L + \Delta L^2) - (r^2 - 2 \cdot r \cdot h_1 + h_1^2)} = \sqrt{\Delta L \cdot (2 \cdot r + \Delta L) + h_1 \cdot (2 \cdot r - h_1)} \tag{5b}$$

$$R_{P\_h1} = r_1 = \sqrt{\Delta L \cdot (2 \cdot r + \Delta L) + h_1 \cdot (2 \cdot r - h_1)} \tag{5c}$$

where:

R_{P_h1} is protection radius in real ground level [m]

r_1 is horizontal distance between center of rolling sphere S_1 and rod tip S_0 in height h_1 [m]

h_1 is total height of rod tip above the ground [m]

Based on Fig. 4, equation (5c) is derived correctly, however, this equation is valid only on ground level (height h_1 is taken into consideration). The equation isn't valid for any vertical distance (at least 5m) from rod tip as it is mentioned in [1]. It would be only valid if the worst place of leader occurrence (represented by touching rolling sphere with ground) weren't taken into account; this means that rolling sphere is lifted to the height where protection radius is calculated. However, using equation (5c) in this way is totally incorrect because the smallest protected volume of rod is neglected, thus the volume is artificially increased.

Standard [1] uses the equation (5c) exactly the same way as it is mentioned above. It determines protection radius in different heights as follows (Fig. 4):

1. ground level:
   - rolling sphere is touching the real ground (point A_2) and protected spherical volume around the rod tip (point B_2),
   - protection radius is equal to the horizontal distance between points A_2 and S_0,
2. height $h_1 - h_1'$:
   - rolling sphere is lifted above the real ground in such a way that it is touching the imaginary ground' (point A_2') and protected spherical volume around the rod tip (point B_2'),
   - protection radius is equal to the horizontal distance between points A_2' and S_0,
3. height $h_1 - h_1''$:
   - rolling sphere is lifted above the real ground in such a way that it is touching the imaginary ground'' (point A_2'') and protected spherical volume around the rod tip (point B_2''),
   - protection radius is equal to the horizontal distance between points A_2'' and S_0.

It is clear that rolling sphere is moving along the circle with radius equal to $\Delta L + r$ and with center in rod tip (point S_0) until the height is 5m under the rod tip. In that height standard [1] changes the way of protection radius calculation – it uses triangle similarity ($S_0A_{2\_5m}$'C_{2_5m}' and $S_0A_{2\_h}$'C_{2_h}' in Fig. 5) in the height from 5m to 2m under the rod tip. New formula of protection radius calculation shown in [1] is derived as follows:

$$\frac{R_{P\_h}}{R_{P\_5m}} = \frac{h}{5} \tag{6a}$$

$$R_{P\_h} = \frac{h}{5} \cdot R_{P\_5m} \tag{6b}$$

where:

R_{P_h} protection radius in vertical distance $h \in$ <2m,5m> from ESE rod tip [m]

R_{P_5m} protection radius calculated according to (5c) in vertical distance h =5m from ESE rod tip [m]

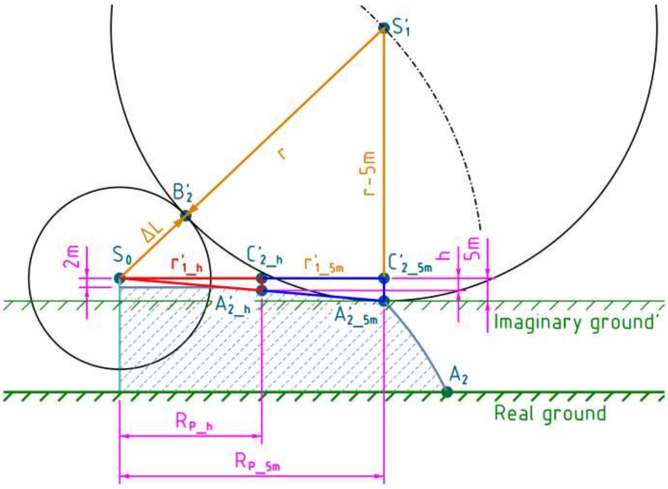

Fig. 5 Calculation of protected volume in vertical distance $h \in$<2m,5m> from rod tip using the triangle similarity according to [1]

Based on formulas shown above (5c), (6b) three-dimensional protected volume of ESE lightning rod can be visualized. Inputs of calculation are these (Fig. 6, Fig. 7):

- lightning protection level LPL IV: $r = 60$m,
- height of ESE lightning rod above the ground: $h = 25$m,
- length increase of upward leader:
  - a) $\Delta L = 20$m,
  - b) $\Delta L = 0$m.

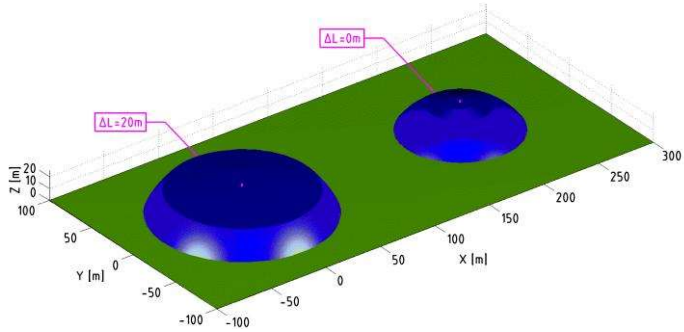

Fig. 6. Three-dimensional volume of protection of ESE lightning rod calculated according to [1]

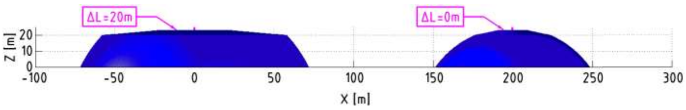

Fig. 7. Cross-section (plane X-Z) of protected volume of ESE lightning rod calculated according to [1]

3.2 CORRECT DERIVATION OF PROTECTION RADIUS OF ESE LIGHTNING ROD

The worst case of downward leader tip occurrence (point S_1) must be taken into consideration just like in the case of conventional lightning rod. In the case of ESE rod this location is defined by point of touch of rolling sphere with real ground (point A_2) and with sphere created by length increase of upward leader (point B_2).

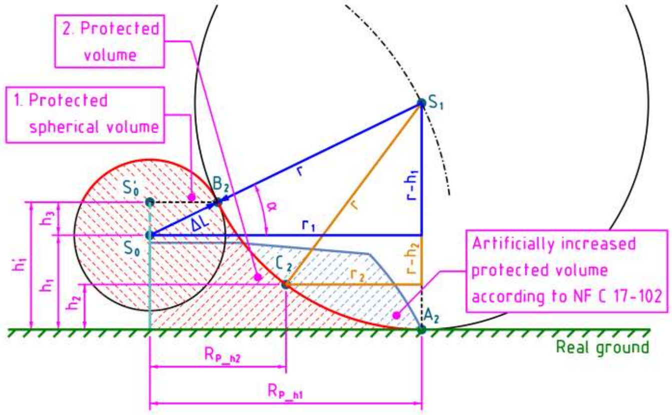

Fig. 8 Cross-section (plane X-Z) of real protected volume of ESE lightning rod

Based on Fig. 8 real protection radius of ESE lightning rod in different heights must be calculated according to formulas shown below:

1. real ground level (height h_1) - R_{P_h1}:
   - formula (5c),
2. any height above real ground level (for instance height h_2) - R_{P_h2}:
   - formula (3c).

The only difference between protected volume of ESE lightning rod determined by formulas shown above and volume of protection of conventional rod is that in case of ESE lightning rod protection radius is determined up to height h_1' ($h_1' \geq h_1$). This height is defined by point of touch of rolling sphere with sphere created by length increase of upward leader (point B_2). The following formulas shows how that height is derived:

$$\cos\alpha = \frac{r_1}{\Delta L + r} \tag{7a}$$

$$h_3 = \sin\alpha \cdot \Delta L \tag{7b}$$

$$h_1' = h_1 + h_3 \tag{7c}$$

where:

h_1' is total height of ESE lightning rod tip above the ground increased by height increase h_3 [m]

Based on formulas (3c), (5c) real three-dimensional model of protected volume of ESE lightning rod can be visualized. Inputs of calculation are shown below (Fig. 9, Fig. 10):

- lightning protection level LPL IV: $r = 60$m,
- height of ESE lightning rod above the ground: $h = 25$m,
- length increase of upward leader:
  - a) $\Delta L = 20$m,
  - b) $\Delta L = 0$m.

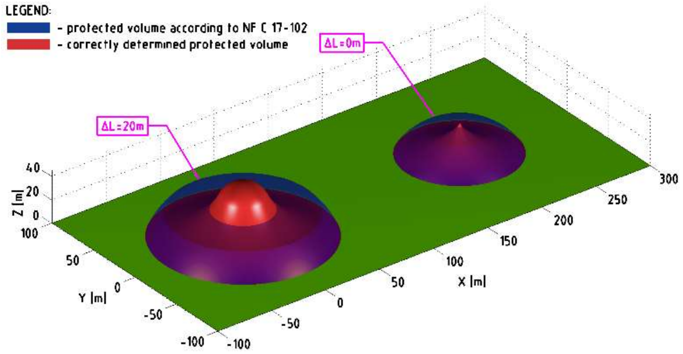

Fig. 9 Real three-dimensional volume of protection of ESE lightning rod

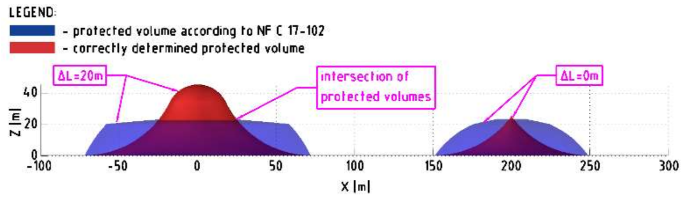

Fig. 10 Cross-section (plane X-Z) of real protected volume of ESE lightning rod

4 PRACTICAL COMPARISON BETWEEN CORRECT CALCULATION OF PROTECTION RADIUS AND CALCULATION ACCORDING TO NF C 17-102

Based on the comparison between equation (3c) and (5c) it is clear that protection radius calculated according to chapter 3.2 will be always smaller than radius calculated in compliance with standard [1] for any height. This is caused by subtracting horizontal distance r_2 from protection radius on ground level R_{P_h1} in equation (3c) while no subtracting is taken into consideration in equation (5c). The protection radii will only be equal on ground level and in point of intersection of protected volumes. Total percentage decrease of protection radius ΔR_P will depend on every variable in equation (3c). Based on this, three cases can be created in such a way that only one variable will change and others will stay constant:

1. case 1: $h_1 = var.$, $r = const.$, $\Delta L = const.$,
2. case 2: $r = var$, $h_1 = const.$, $\Delta L = const.$,
3. case 3: $\Delta L = var$, $h_1 = const.$, $r = const.$.

Total percentage decrease of protection radius ΔR_P shown in figures (Fig. 11, Fig.12, Fig. 13) is calculated as follows:

$$\Delta R_P = 100 - \frac{100 \cdot R_{P\_h\_correct}}{R_{P\_h\_wrong}} \qquad (8)$$

where:

$R_{P\_h\_correct}$ is protection radius of ESE lightning rod in any height determined according to chapter 3.2 [m]

$R_{P\_h\_wrong}$ is protection radius of ESE lightning rod in any height determined according to [1] or chapter 3.1 respectively [m]

Curves shape of total percentage decrease is similar for all considered cases (Fig. 11, Fig. 12, Fig. 13). Once the height h_2 increases, total percentage decrease of protection radius increases as well. Maximal decrease will be reached in most cases ($h_1 < 48$m) in height where the way of determining protection radius is changed (chapter 3.1 – triangle similarity). In case that $h_1 \geq 48$m, maximal decrease will be reached in even smaller height (for instance: if $h_1 = 50$m, $\Delta R_{P\_max} \cong 60\%$ will be reached in height $h_2 = 42$m). With further increase of height h_2, percentage decrease will be decreasing due to change of protection radius determining in accordance with [1]. The total percentage decrease of protection radius will be zero in point of intersection of protected volumes due to equal protection radii.

4.1 CASE 1: $h_1 = var.$, $r = const.$, $\Delta L = const.$

Following inputs were taken into account:

- lightning protection level LPL IV: $r = 60$m,
- total height of rod tip above the ground: $h_1 \in \langle 10\text{m}|60\text{m}\rangle$ with step of 10m,
- length increase of upward leader: $\Delta L = 25$m.

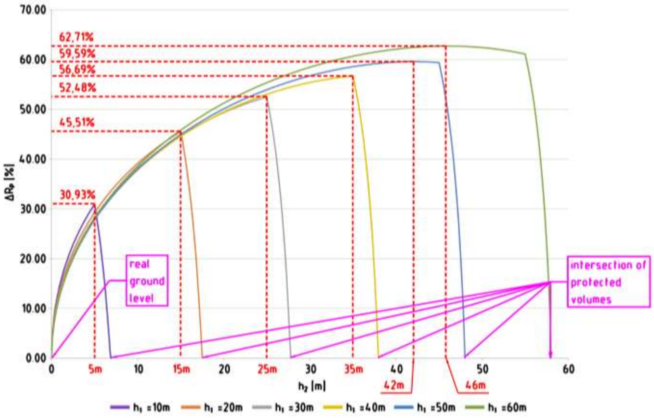

Fig. 11 Total percentage decrease of protection radius ΔR_P depending on different height h_2 in case 1 ($h_1 = var.$, $r = const.$, $\Delta L = const.$)

Based on Fig. 11 it is obvious that if height of rod tip above the ground h_1 increases, percentage decrease of protection radius ΔR_P increases as well. In this case, maximal decrease $\Delta R_{P\_max} \cong 63\%$ ($h_2 = 46$m) will be reached in height $h_1 = 60$m. Otherwise, minimal decrease $\Delta R_{P\_min} \cong 31\%$ ($h_2 = 5$m) will be reached in height $h_1 = 10$m.

4.2 CASE 2: $r = var.$, $h_1 = const.$, $\Delta L = const.$

Following inputs were taken into account:

- lightning protection level LPL I, II, III, IV: $r \in \{20\text{m}|30\text{m}|45\text{m}|60\text{m}\}$,
- total height of rod tip above the ground: $h_1 = 20$m,
- length increase of upward leader: $\Delta L = 25$m.

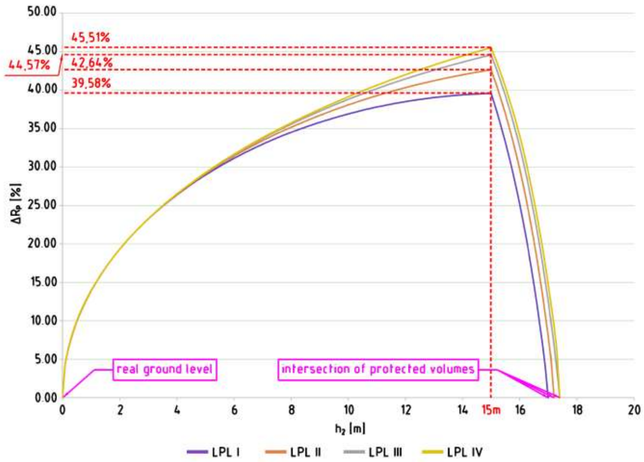

Fig. 12 Total percentage decrease of protection radius ΔR_P depending on different height h_2 in case 2 ($r = var.$, $h_1 = const.$, $\Delta L = const.$)

Based on Fig. 12 it is obvious that if lightning protection level LPL decreases (radius of rolling sphere increases), percentage decrease of protection radius ΔR_P increases. In this case, maximal decrease $\Delta R_{P\_max} \cong 46\%$ ($h_2 = 15$m) will be reached in LPL IV. Minimal decrease $\Delta R_{P\_min} \cong 40\%$ ($h_2 = 15$m) will be reached in LPL I.

4.3 CASE 3: $\Delta L = var.$, $r = const.$, $h_1 = const.$

Following inputs were taken into account:

- lightning protection level LPL IV: $r = 60$m,
- total height of rod tip above the ground: $h_1 = 60$m,
- length increase of upward leader: $\Delta L \in \langle 10\text{m}|60\text{m}\rangle$ with step of 10m.

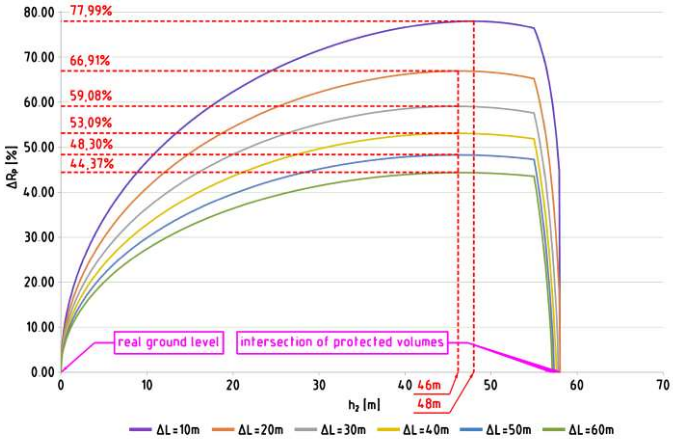

Fig. 13 Total percentage decrease of protection radius ΔR_P depending on different height h_2 in case 3 ($\Delta L = var.$, $r = const.$, $h_1 = const.$)

Based on Fig. 13 it is obvious that if length increase of upward leader decrease, percentage decrease of protection radius ΔR_P increases. In this case, maximal decrease $\Delta R_{P\_max} \cong 78\%$ ($h_2 = 48$m) will be reached in $\Delta L = 10$m. Minimal decrease $\Delta R_{P\_min} \cong 44\%$ ($h_2 = 46$m) will be reached in $\Delta L = 60$m.

5 CONCLUSION

This article doesn't doubt unproved early streamer emission advantage of ESE lightning rod against the conventional rod. On the contrary, it is trying to implement that effect to rolling sphere method correctly because it is used incorrectly in French standard [1]. For that reason the main aim of that article is to explain why protected volume of ESE lightning rod determined according to [1] is wrong.

The proof of wrong calculation of protection radius according to [1] is as follows:

- if ESE lightning rod with $\Delta L = 0$m is taken into consideration, volume of protection determined according to [1] should be the same as protected volume of conventional rod, however, it is clear from Fig. 10 that the volumes are different.

The main cause of wrong calculation is the formula (5c) which is used for calculation of protection radius in any height in spite of the fact that formula is valid only on ground level. If formula (5c) is used for calculation of protection radius in any height, protected volume of ESE lightning rod will be artificially increased (Fig. 10). This is the reason why volume of protection of ESE lightning rod looks much bigger than the volume of conventional rod, however, they cannot be compared because of different way of protection radius calculation. In case of single conventional rod, rolling sphere always touches the ground and protection radius in any height is calculated as difference between protection radius on ground level (formula (2c) or formula (5c) if $\Delta L = 0$m) and horizontal distance between rolling sphere center and its edge in particular height (formula (3b)).

Correct protected volume of ESE lightning rod should be determined as the volume of conventional rod with one difference - protection radius on ground level should by calculated according to formula (5c), not according to (2c) (Fig. 8).

Based on comparison of protected volumes of ESE lightning rod (chapter 3.1 and 3.2) it is clear that correct protected volume of ESE lightning rod is smaller than the volume defined by the standard [1]. Percentage decrease of protection radius in any height depends on three variables - object height, lightning protection level LPL and length increase of upward leader. In general, the percentage decrease will be bigger if following variables are taken into account:

- higher object (Fig. 11),
- smaller lightning protection level LPL (Fig. 12),
- smaller length increase of upward leader (Fig. 13),

I recommend (mainly for safety reasons) all owners of ESE lightning rods to check its real volume of protection based on the facts mentioned above. There is a high probability that part of large structure will be outside the protected volume if rolling sphere method is used correctly. This is probably one of the reasons why lightning strikes to protected volumes of ESE lightning rods are detected.

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